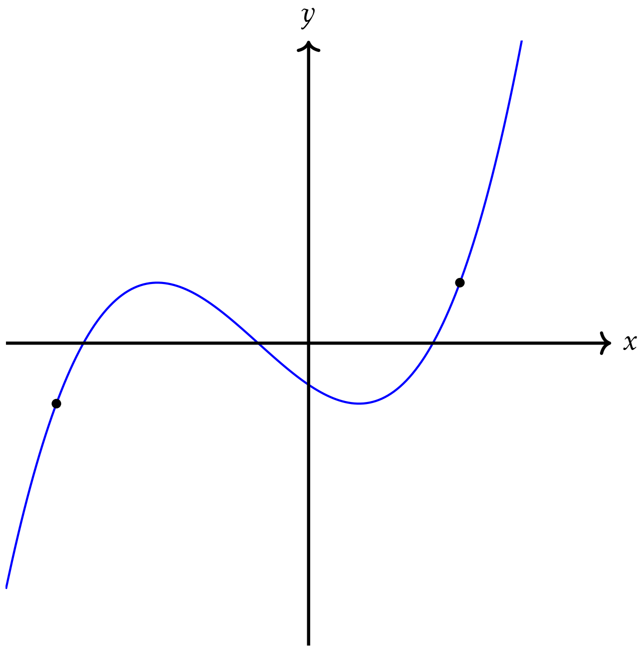
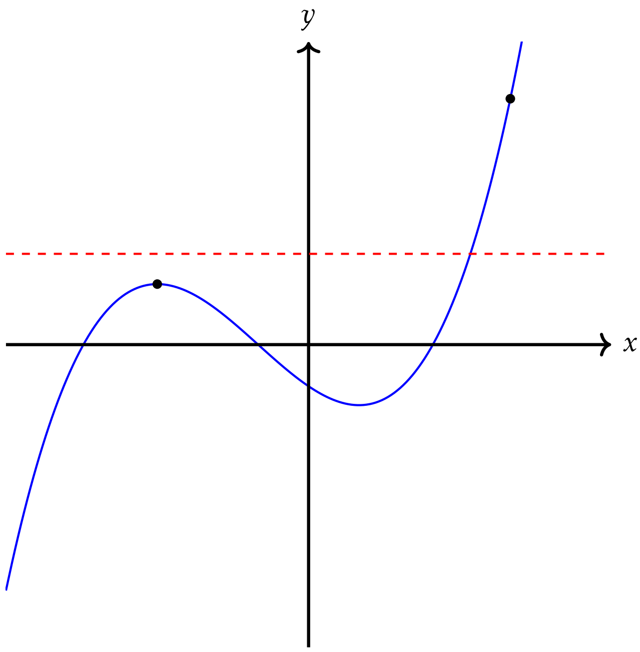
Change of Sign



Fact (Sign-change rule) — If f is a continuous function and if $f(a) < 0$ and $f(b) > 0$ then there is a some value c between a and b such that $f(c) = 0$

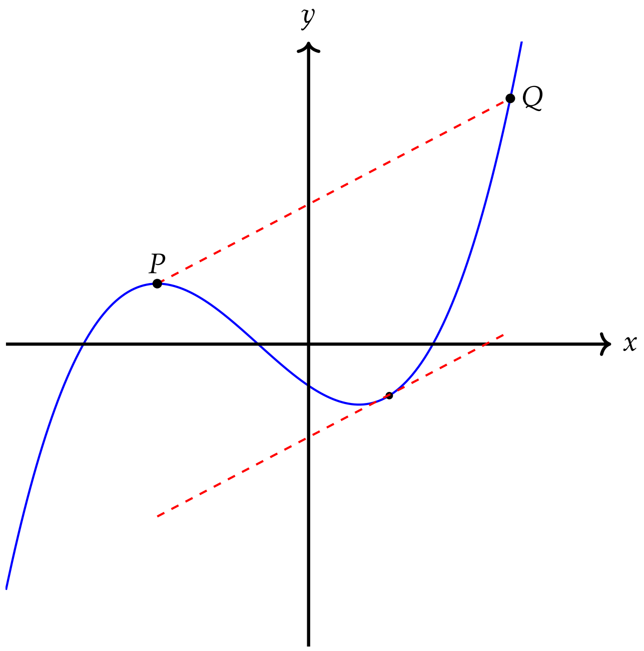
Intermediate Value Theorem



Fact (Intermediate Value Theorem) — If f is a continuous function then for all values $\min(f(a), f(b)) \leq y \leq \max(f(a), f(b))$ there is some value c such that $f(c) = y$

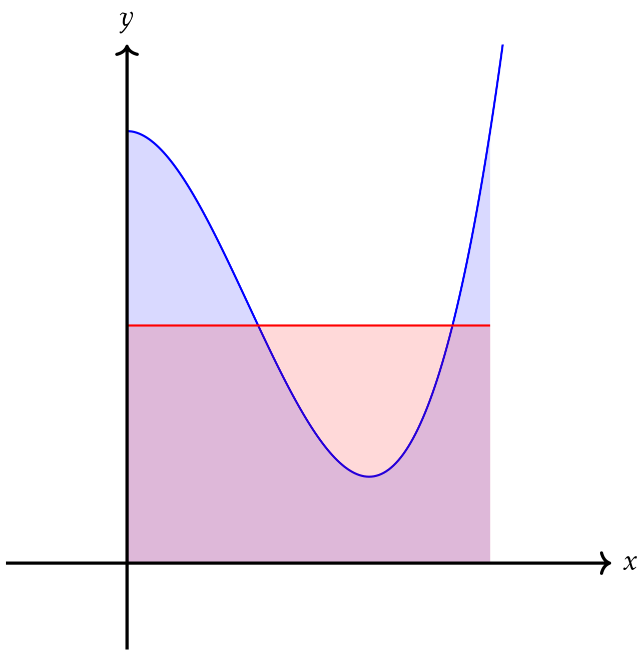
Fact — The sign-change rule is a special case of the intermediate value theorem

Mean Values



Fact — If f is a function with *continuous* derivative, and points $P = (a, f(a))$ and $Q = (b, f(b))$ then there is a point $c \in (a, b)$ with tangent parallel to the line joining the points, ie

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



Definition. The **mean value** of a function on (a, b) is $\frac{1}{b-a}\int_a^b f(x)dx$

Example

Find the mean value of $f(x) = x^3 + 2x + 1$ on $(1,3)$

$$
\begin{aligned}
m &= \frac{1}{3-1}\int_1^3 (x^3 + 2x + 1)dx \\
&= \frac{1}{2}\left[\frac{x^4}{4} + x^2 + x\right]_1^3 \\
&= \frac{1}{2}\left(\left(\frac{81}{4} + 9 + 3\right) - \left(\frac{1}{4} + 1 + 1\right)\right) \\
&= \frac{1}{2}\left(\frac{129}{4} - \frac{9}{4}\right) = \frac{1}{2}\cdot 30 = 15
\end{aligned}
$$

Example

The speed, v metres per second, of a car t seconds after the driver presses the accelerator can be modelled by the equation $v = 25\left(1 - e^{-0.3t}\right)$ for $t \geq 0$. Determine, according to the model, the mean speed of the car over the first 10 seconds after the accelerator is pressed. Give your answer correct to 2 decimal places.

$$
\begin{aligned}
\textit{Average speed} &= \frac{1}{10-0}\int_0^{10} 25\left(1 - e^{-0.3t}\right)dt \\
&= \frac{25}{10}\int_0^{10}\left(1 - e^{-0.3t}\right)dt \\
&= 2.5\left[t + \frac{1}{0.3}e^{-0.3t}\right]_0^{10} \\
&= 25 + \frac{2.5(e^{-3} - 1)}{0.3} \\
&= 17.08 \quad (2\ d.p.)
\end{aligned}
$$